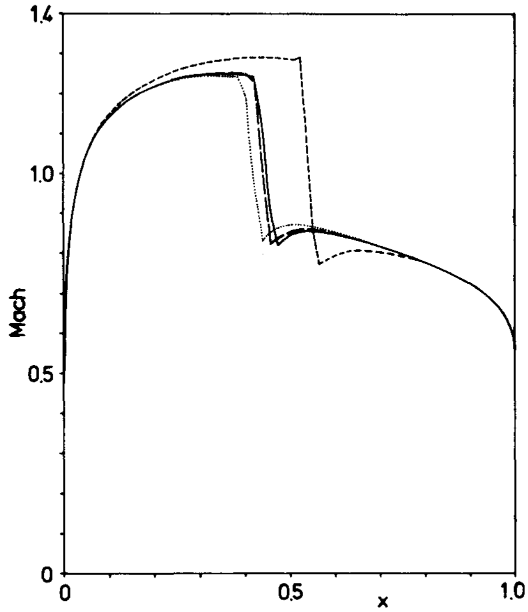

Fig. 4 Mach number distribution along upper surface of NACA-0012 airfoil (M=0.76, α=1 deg). GTC=12.0: ——; GTC=3.6: ---- first iteration, · · · · second iteration, —— third iteration.

3c, for instance in shock position and height of the supersonic bubble.

Finally, we see from the Mach number distribution along airfoil, shown in Fig. 4, how convergence is obtained after a few iterations, the error dying away while alternating in sign.

Conclusion

Replacing the full linear far-field solution by simple analytical relations at subcritical flow boundaries, the numerical computation of transonic flow is concentrated to its nonlinear domain. Thereby, the overall grid size is reduced and we get either a gain in computational time keeping the numerical resolution fixed or a better resolution in almost the same computational time, the number of points being kept fixed.

Numerical solutions to the Euler and Navier-Stokes equations for steady transonic flow embedded in a subcritical far-field flow are usually achieved in time-dependent calculations. The outer boundary conditions are formulated along characteristic lines using Riemann variables.[1] In conjunction with a fast potential flow solver, the present method could be favorably employed to precalculate near-field boundary conditions in the form of Riemann variables for these more sophisticated equations in order to substantially reduce computation time or to improve the numerical resolution of singular flow problems.

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Calibrating Boundary Layers for Suction

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Introduction

THE concept of boundary-layer control by means of boundary-layer suction is a well-known and fundamental tool used by aerodynamicists. The idea of boundary-layer theory and boundary-layer control was first conceived by Prandtl.[1] The work on boundary-layer control started in 1904. Previous work, discussed by Lachmann,[1] pertained to either experimental work or solutions of boundary-layer equations. The results of research by Iglisch,[2] Schlichting,[3] and Rheinboldt[4] have to date not been refined. But, with increase in suction velocity, the boundary-layer assumptions are no longer valid, and some error is introduced in the results. Furthermore, the nature of boundary-layer equations is such that disturbances downstream of the flow are not detected upstream of the disturbance. This poses a serious error in case of suction of the boundary layer; i.e., the fluid is only sucked into the slot from regions near the suction surfaces, and no fluid is sucked in from upstream of the suction region. For this reason, a technique was devised by which calibration of boundary-layer equations could be achieved, using a numerical solution of complete incompressible Navier-Stokes equations, which gives results accurate enough to be called an exact solution.

Mathematical Model

The governing equations for two-dimensional incompressible steady flow without body force can be written as

$$\nabla^2\psi+\omega=0 \tag{1}$$

$$\nabla^2\omega+Re\left(\frac{\partial\psi}{\partial y}\cdot\frac{\partial\omega}{\partial x}-\frac{\partial\psi}{\partial x}\cdot\frac{\partial\omega}{\partial y}\right)=0 \tag{2}$$

where ω is the transverse vorticity component defined by

$$\omega=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$$

the stream function ψ is defined by

$$u=\frac{\partial\psi}{\partial y}\qquad v=-\frac{\partial\psi}{\partial x}$$

and $Re=UL/\nu$.

In Eqs. (1) and (2) the freestream velocity U and the length L are used as reference quantities. In the case of slot

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Table 1 Influence of suction

$-v_0^*$	x for $H=2.5$	δ_1	x Blasius for same δ_1	Δx	$\Delta x/x$
1.0	1.39	1.882	$(1.882/1.73)^2 = 1.183$	0.207	14.9
1.5	1.45	1.844	$(1.844/1.73)^2 = 1.136$	0.314	21.7
3.0	1.575	1.775	$(1.775/1.73)^2 = 1.053$	0.522	33.1

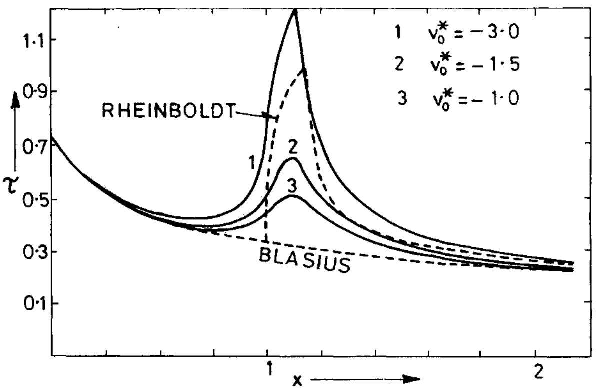

Fig. 1 Wall shear variation.

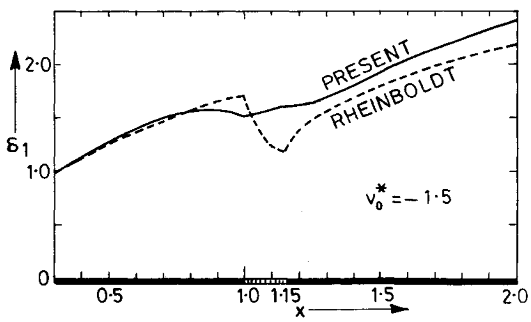

Fig. 2 Displacement thickness variation.

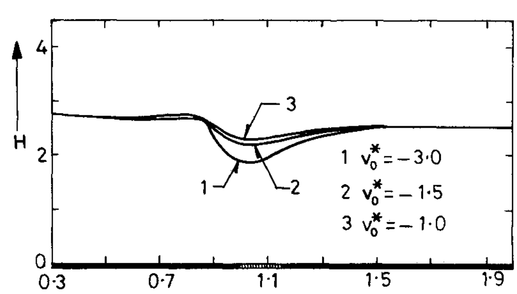

Fig. 3 Shape factor variation.

suction, L is taken as the distance from the leading edge of the plate to the beginning of the suction through a porous slot.

Since the equations are elliptic, boundary conditions are prescribed on an enclosing surface. For the plate, difficulty arises at the downstream boundary condition. The problem is overcome using extrapolation[5] at a suitable downstream distance. Equations (1) and (2) are solved by using appropriate boundary conditions.[6-8]

The flowfield is divided into a rectangular grid, and the equations are written in their finite-difference form. The

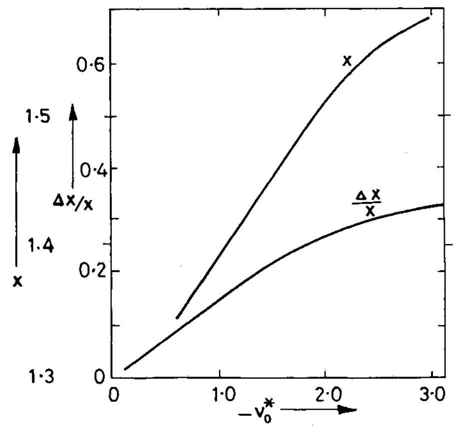

Fig. 4 Displacement of origin.

finite-difference equations are solved for ψ and ω at the nodal points by a modified version of extrapolated Liebmann's method (SOR). In the stream function equation the derivatives are replaced by central differences, and in the vorticity equation the second-order derivatives are replaced by central differences, whereas the first-order derivatives are replaced by forward or backward differences depending on whether the coefficient of these derivatives is positive or negative; also, first-order derivatives of stream function are replaced by central differences. Relaxation continues until changes in values of ψ and ω are less than 10^{-6}.

Results and Discussion

Results are presented for $Re \sim 10^4$ and the suction parameter $v_0^* = (v_0/U)\sqrt{UL/\nu} = -1.0$, -1.5, and -3.0. Figure 1 shows the comparison of wall shear with those of Rheinboldt[2] and Blasius. There is a general disagreement in the slot region, which is to be expected. The upstream influence of increased suction on wall shear, as well as the increased upstream region of influence is shown. The region of influence of slot suction is observed by comparison with Blasius' value of wall shear. Figure 2 shows a comparison of displacement thickness. The sharp changes in displacement thickness obtained by Rheinboldt[2] are not observed in the present case.

The "shape factor," which to an extent represents the stability of flow, is shown in Fig. 3 for $v_0^* = -1.0$, -1.5, and -3.0. Lower values of shape factor indicate higher stability of flow, and it is observed that because of suction, the velocity profile becomes more stable.

In the vicinity of the slot, the boundary layer ceases to be similar (see Fig. 4); the similarity is regained at some downstream distance. It might be of interest to relate this distance to the suction parameters. This is accomplished by defining the nearly similar profile as the location where the shape factor is 2.5 (4% deviation from Blasius). These locations are $x = 1.39$, 1.45, and 1.575 for $-v_0^* = 1.0$, 1.5, and 3.0. The corresponding velocity profiles are identical to Blasius profiles, with the virtual origins shortened by 0.207, 0.314, and 0.522 (see Table 1). Figure 4 shows $\Delta x/s$ vs $-v_0^*$ and x vs $-v_0^*$, where Δx is the origin displacement and x is the location where the profiles over the plate with the slot are nearly recovered. Such results are not possible with the use of boundary-layer equations.

Conclusion

It is observed that influence of suction is felt upstream of the porous slot. The results therefore differ from boundary-layer solutions; however, these differences in the solution are not made up even on the downstream side. The virtual origin due to suction is established, and the relation is established on displacement of origin with the suction parameter.

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Transition from Laminar to Turbulent Flow in Separated Shear Layers

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Nomenclature

c = airfoil chord
L = streamwise distance from separation to transition
ℓ = unit of length
M = Mach number
Re = Reynolds number
s = wetted length from stagnation point to boundary-layer separation
T' = relative turbulence intensity in freestream
U = velocity
ν = kinematic viscosity

Subscripts

c = based on length c
L = based on length L
s = based on local value at separation
∞ = based on freestream conditions

Introduction

WHEN a laminar boundary layer separates from a solid boundary, the free shear layer may undergo transition to the turbulent state and the turbulent layer may subsequently reattach to the solid boundary. In such cases, the region of separation is referred to as a separation bubble. Laminar-to-turbulent boundary-layer transition on many practical aerodynamic and hydrodynamic configurations occurs by this process. Therefore, there is interest in both the prediction of laminar separation and the streamwise distance from separation to transition. The location of separation may be predicted by semiempirical criteria or by various computer codes written for calculating laminar boundary-layer characteristics. This Note concerns the streamwise length from separation to "transition," denoted by L or a Reynolds number $(U/\nu)_s L = Re_{sL}$. Because transition lengths in separation bubbles are relatively short compared to, for example, airfoil chord, authors writing on this subject usually have not made a distinction between the beginning and end of transition.

There have been proposals for methods of predicting Re_{sL}. Several of the semiempirical types are briefly reviewed in Ref. 1, where new data from subsonic wind tunnel experiments are presented. References 2 and 3 discuss separation bubbles and describe efforts at predicting transition based upon the e^n approach, whereby an unstable shear layer is assumed to undergo transition through the amplification of Tollmein-Schlichting waves. Reference 4 is a recent analysis of the problem of computing the flow in the bubble region. The present Note shows that a feature of the transition phenomenon hidden in the data of Ref. 1 and apparently not recognized in earlier investigations should be considered in future work.

Technical Discussion

The purpose of this Note is to point out a rather strong factor that seems to have been generally overlooked in the context of transition over separation bubbles or other free shear layers. This factor, which is commonly called the unit Reynolds number Re/ℓ or (U/ν), is shown in this Note to exert an influence very similar to its usual effect on the transition of bounded shear layers (see, e.g., Refs. 5–7).

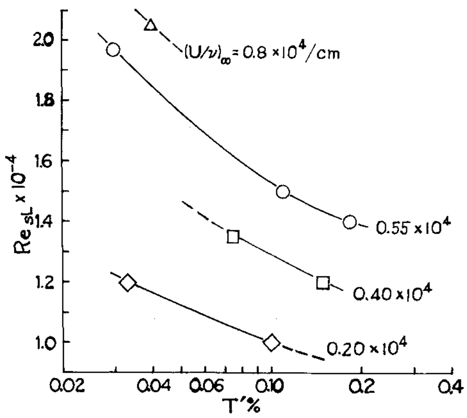

Fig. 1 Experimental data of Ref. 1 recast to indicate the relationship of Re_{sL} and T' when Re/ℓ is constant.

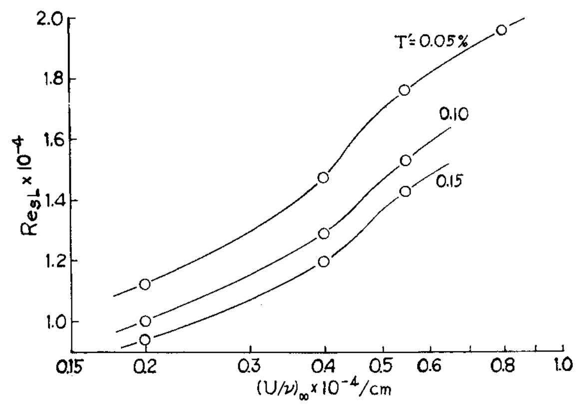

Fig. 2 The influence of Re/ℓ upon Re_{sL} in the subsonic separation-bubble flows of Ref. 1.

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